COOPERATIVE DIVERSITY IN WIRELESS NETWORKS

Andreas Meier, John S. Thompson
Institute for Digital Communications, School of Engineering and Electronics
The University of Edinburgh, Edinburgh, EH9 3JL, UK.
Email: andreame@ee.ethz.ch, John.Thompson@ed.ac.uk

ABSTRACT

An ad-hoc network with a base station, a mobile and a third station acting as a relay is analysed. The channels are modelled using path loss, Rayleigh fading and thermal noise. Different combining methods and diversity protocols are compared. In the simulations, the amplify and forward protocol shows a better performance than the decode and forward protocol. To combine the incoming signals the channel quality should be estimated as well as possible. Information about the average quality shows performance benefits, and a rough approximation about the variation of the channel quality increases the performance even more. Whatever combination of diversity protocol and combining method is used, second level diversity is observed. The relative distances between the relay and the stations has a large effect on the performance.

I. INTRODUCTION

In wireless transmission the signal quality suffers occasional, severe degradations due to effects like fading caused by multipath propagation. To reduce such effects, diversity, as proposed by [1], can be used to transfer the different samples of the same signal over essentially independent channels. There are several approaches to implement diversity in a wireless transmission. Multiple antennas can be used to achieve space diversity. But multiple antennas can not always be implemented on small terminals or the destination is just too far away to get a good signal quality, especially when power consumption is an important criteria. An other interesting approach, as proposed in [2] might be to build an ad-hoc network using another mobile station as a relay.

In such a system combinations of several relaying protocols and different combining methods are examined to see their effects on the performance. The transmission protocols used in this paper are *Amplify and Forward* and *Decode and Forward.* In the simulation these can both be seen to achieve full diversity as was proved in [2]. Basically three different types of combining methods are examined which differs in the knowledge of the channel quality they need to work.

One combination that achieves a good performance is then used to see the effect on the performance depending on the location of the relay. This information is crucial to decide when a mobile relay should be used.

II. SYSTEM MODEL

To achieve diversity in a single user wireless system a third mobile acting as a relay is used. The sender S, transmits the data to the destination D, while the relay R is listening to this transmission. The relay sends the received data burst after processing to the destination as well, where the two received signals are combined. As proposed in [2], orthogonal channels are used for the two transmissions. Without loss of generality, this can be achieved using time division multiple access channels, which is done in all the simulations in this work.

The transferred data is a random bipolar bit sequence which is either modulated with *Binary Phase Shift Keying* (BPSK) or *Quadrature Phase Shift Keying* (QPSK). QPSK consists of two independent (orthogonal) BPSK systems and therefore has double the datarate compared to BPSK. This fact can be used to compare the performance of a single link channel compared to one using diversity. While the single link channel uses BPSK the diversity channel, which has to send the data twice, modulates the channel with QPSK which results in the same overall bandwidth for both systems. Note that both systems will use the same amount of transmission power.

The channel is modelled considering thermal noise $z_{s,d}[n]$ (additive complex Gaussian noise), path loss and Rayleigh (block-)fading. For the amplitude path loss $d_{s,d} \propto \frac{1}{R^2}$ is assumed which is modelled by the plane-earth model. The fading coefficient $a_{s,d}[n]$ is modelled as a zero mean, complex Gaussian random variable with variances $\sigma^2_{s,d}$.

$$y_d[n] = d_{s,d}a_{s,d}[n]x_s[n]+z_{s,d}[n] = h_{s,d}[n]x_s[n]+z_{s,d}[n] \quad (1)$$

In (1) s, d denote the sender and the destination, $x_s[n]$ is the transmitted symbol and $y_d[n]$ the received symbol. The scalar $h_{s,d}[n] = d_{s,d}a_{s,d}[n]$ represents the overall attenuation.

There are two popular implementations to transmit over a wireless network. One is the simple direct link which sends the data only once. The other is the two sender arrangement which sends the data twice over different antennas. These two standard implementations put the performance of the arrangements used in this work into perspective.

The error probability of a single link transmission is as shown in [1]

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right) \quad (2)$$

where $\bar{\gamma}_b$ denotes the average signal-to-noise ratio, defined as $\bar{\gamma}_b = \frac{\xi}{2\sigma^2}E(a^2)$, where $E(a^2) = a^2$.

The performance of a two sender transmission with MRC at the receiver can be expressed [1] as

$$P_b = \frac{1}{4}(1-\mu)^2(2+\mu) \qquad \mu = \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}. \tag{3}$$

III. DIVERSITY PROTOCOLS

The cooperative transmission protocols used in the relay station are either *Amplify and Forward* (AAF) or *Decode and Forward* (DAF). These protocols describe the processing of the received data at the relay station before the data is sent to the destination.

A. Amplify and Forward (AAF)

This method is often used when the relay has only limited computing time/power available or the time delay, caused by the relay to de- and encode the message, has to be minimised. The signal received by the relay was attenuated and needs to be amplified before it can be sent again. In doing so the noise in the signal is amplified as well, which is the main drawback of this protocol.

The incoming signal is amplified block wise. Assuming that the channel characteristic can be estimated perfectly, the gain for the amplification can be calculated as follows.

The power of the incoming signal (1) is given by

$$\begin{aligned} E[|y_r^2|] &= E[|h_{s,r}|^2]E[|x_s|^2] + E[|z_{s,r}|^2] \\ &= |h_{s,r}|^2\xi + 2\sigma_{s,r}^2, \end{aligned} \tag{4}$$

where s denotes the sender, r the relay and $\xi = E[|x_s|^2]$ denotes the energy of the transmitted signal. To send the data with the same power the sender did, the relay has to use a gain β of

$$\beta = \sqrt{\frac{\xi}{|h_{s,r}|^2\xi + 2\sigma_{s,r}^2}} \tag{5}$$

This term has to be calculated for every block and therefore the channel characteristic of every single block needs to be estimated.

B. Decode and Forward (DAF)

A wireless transmission typically uses digital modulation and the relay has enough computing power, so DAF is most often the preferred method to process the data in the relay. The received signal is first decoded and then re-encoded. So there is no amplified noise in the sent signal, as is the case using a AAF protocol. There are two main implementations of such a system.

The relay can decode the original message completely. This requires a lot of computing time, but has the advantage that an error correcting code could be processed in the relay. If the relay station has not the computer power or is not allowed to fully decode the message, the incoming signal can just be decoded and re-encoded symbol by symbol. So there is no amplified noise in the sent signal, as it is the case using a AAF protocol. Within this work this second approach is used to get an idea about the raw performance of the DAF protocol.

IV. COMBINING METHODS

As soon as there is more than one incoming transmission with the same burst of data, the incoming signals have to be combined. In this work the signals are combined only with the current information of the signal and channel. The four used combining methods differs in the knowledge about the channel quality they need to work.

A. Equal Ratio Combining (ERC)

This is the most simplest combining method, which should only be used if there is no information about the channel quality available or the computing capacity is extremely limited. The incoming signals are just added up before the symbols are detected. Note that you do not need information about the quality but about the phase shift of the signal which occurs due to fading.

$$y_d[n] = y_{s,d}[n] \cdot e^{-\angle h_{s,d}[n]} + y_{r,d}[n] \cdot e^{-\angle h_{r,d}[n]} \tag{6}$$

The parameters $y_{s,d}[n]$ and $y_{r,d}[n]$ denote the incoming signal from the sender and the relay.

B. Fixed Ratio Combining (FRC)

A much better performance can be achieved when the incoming signals are weighted with a constant ratio, which will not change a lot during the whole communication. The ratio which is described with the parameters $d_{s,d}$ and $d_{r,d}$ should represent the average channel quality and therefore should not take account of temporary influences on the channel due to fading or other effects. In this work no algorithm is used to estimate the optimal ratio. Instead, the best ratio is approximated by simulating different values to get an idea about the potential of this combining method.

$$\begin{aligned} y_d[n] = d_{s,d} \cdot y_{s,d}[n] \cdot e^{-\angle h_{s,d}[n]} + \\ + d_{r,d} \cdot y_{r,d}[n] \cdot e^{-\angle h_{r,d}[n]} \end{aligned} \tag{7}$$

C. Signal to Noise Ratio Combining (SNRC)

An even better performance can be achieved when precise information about the current state of the different channels is known. An often used value to characterise the quality of a link is the SNR, which is used to weight the received signals.

$$\begin{aligned} y_d[n] = \mathrm{SNR}_{s,d} \cdot y_{s,d}[n] \cdot e^{-\angle h_{s,d}[n]} + \\ + \mathrm{SNR}_{r,d} \cdot y_{r,d}[n] \cdot e^{-\angle h_{r,d}[n]} \end{aligned} \tag{8}$$

The estimation of the SNR of a multi-hop link using AAF or a direct link can be performed by sending a known symbol sequence in every block. This sequence is used to estimate the phase shift as well. If the multi-hop link is using a DAF protocol the receiver can only see the channel quality of the last hop. It is assumed that the relay sends some additional information about the quality of the unseen hops to the destination, so the SNR of the multi-hop link can be estimated. Whatever protocol is used, an additional sequence needs to be sent to estimate the channel quality. This results in a certain loss of bandwidth. In the Appendix A it is shown how the SNR is estimated in the simulation.

D. Enhanced Signal to Noise Ratio Combining (ESNRC)

Another plausible combining method is to ignore an incoming signal when the data from the other incoming channels have a much better quality. If the channels have more or less the same channel quality the incoming signals are rationed equally. In a system where just one relay is used this can be expressed as

$$y_d[n] = \begin{cases} y_{s,d}[n]e^{-\angle h_{s,d}[n]} & (\mathrm{SNR}_{s,d}/\mathrm{SNR}_{s,r,d} > 10) \\ y_{s,r,d}[n]e^{-\angle h_{s,r,d}[n]} & (\mathrm{SNR}_{s,d}/\mathrm{SNR}_{s,r,d} < 0.1) \\ y_{s,d}[n]e^{-\angle h_{s,d}[n]} + & \\ + y_{s,r,d}[n]e^{-\angle h_{s,r,d}[n]} & \text{otherwise} \end{cases} \quad (9)$$

Using this combining method, the receiver does not have to know the channel characteristic exactly. An approximation of the channel quality is enough to combine the signals. Notice that the estimation of the phase shift still needs to be as exact as possible.

V. KEY RESULTS

In this section a combination of different combinations methods and diversity protocols are analysed to illustrate their potential benefits. In a first part, it is assumed that the three stations (sender, relay and destination) have an equal distance from each other. In a second part, the location of the relay station is varied to see the effect on the performance for different locations of the relay. In the following table the abbreviations used in the illustrations are described.

ERC	Equal Ratio Combining (6)
FRC x:y	Fixed Ratio Combining (7) x: Weight of the direct link y: Weight of the multi-hop link
SNRC	SNR Combining (8)
ESNRC	Enhanced SNR Combining (9)
AAF	Amplify and Forward
DAF	Decode and Forward
Distance x:y:z	x: Distance betw sender and destination y: Distance betw sender and relay z: Distance betw relay and destination

TABLE I
ABBREVIATIONS USED IN THE FOLLOWING FIGURES.

A. Equidistant Arrangement

The three stations are arranged at the edges of a triangle with a relative length of one. Hence all the channels will have the same path loss and therefore the same average signal-to-noise ratio.

In Fig. 1 the effect on the performance of the different combining types using an AAF protocol can be seen. The BPSK single link transmission (2) should demonstrate if there is any benefit at all using diversity, while the QPSK two senders link (3) indicates a lower bound for the transmission.

The first result is that whatever combining type is used, the AAF diversity protocol achieves a benefit compared to the direct link. Even the equal ratio combining shows advantages. But

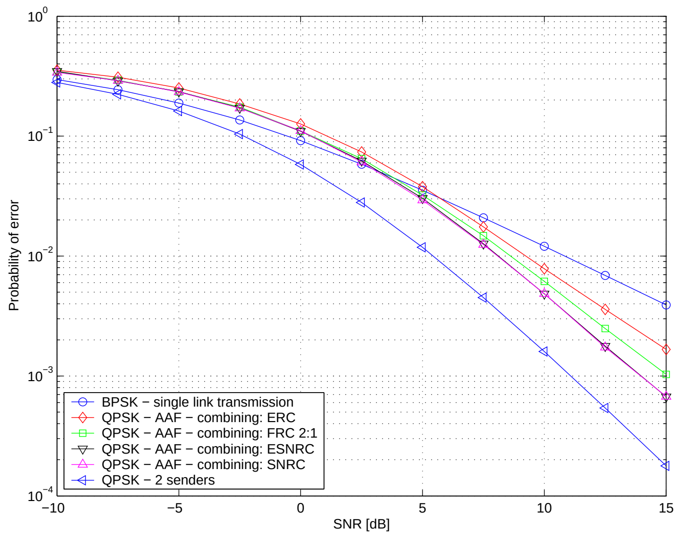

Fig. 1. The different combining types are compared with each other. The best performance results when using SNRC/ESNRC.

compared to the fixed ratio combining, the performance looks quite poor. In addition if an intelligent algorithm to calculate the ratio[1] is used, no bandwidth is wasted in other words the bandwidth is the same than using ERC.

The signal-to-noise ratio combining and the enhanced signal-to-noise ratio combining show roughly the same performance. Remember that for the ESNRC a roughly estimated channel quality for every single block is sufficient in contrast to the SNRC, which needs exact information of the channel quality for every single block, this is a surprising result. It means that the transferred signal in an AAF system contains some information that allows correcting a small difference in the channel quality.

The performance of the combining methods, which have precise information about every single block, is just about one decibel better in SNR than the one using FRC which has just average knowledge of the channel quality. Hence using the AAF protocol, there is no point in wasting a lot of computing power and bandwidth to get exact channel information.

Fig. 2 illustrates the performance of the AAF diversity protocol compared with the DAF protocol. The surprising result is that the AAF diversity protocol always results in a better performance than the DAF protocol whatever combining type is used.

Using equal ratio combining results in a big difference between the two protocols. While the one using AAF shows a quite good performance already, the one using DAF does not have any benefits compared to the BPSK single link. The reason is that a wrong detected symbol at the relay station is really difficult to correct at the destination, where the two incoming signals are combined. Hence an incorrectly detected symbol at the relay station will have a fifty percent probability of also being incorrectly detected at the destination.

This stands in contrast with the equal ratio combining in a system using AAF. Instead of detecting the symbol at the relay, it is amplified and transmitted to the sender. Normally a symbol

[1] In this work the optimal ratio 2:1 was approximated by comparing the performances using different ratios.

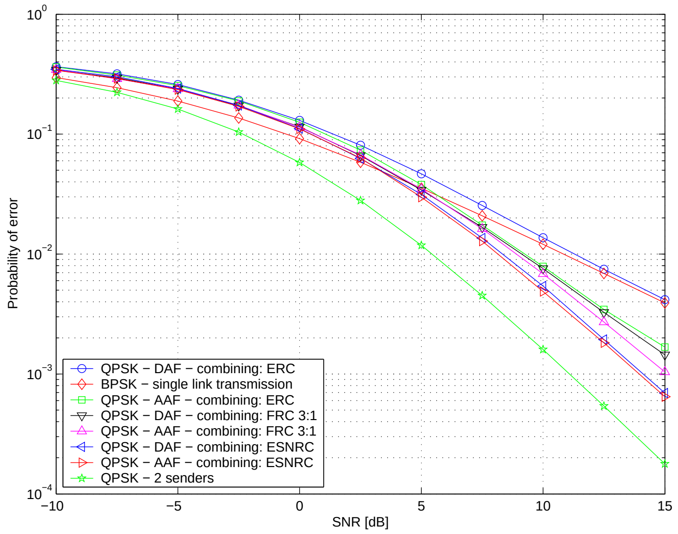

Fig. 2. The two diversity protocols (AAF and DAF) are compared with each other. Notice the different (optimal) FRC-ratios using either AAF or DAF.

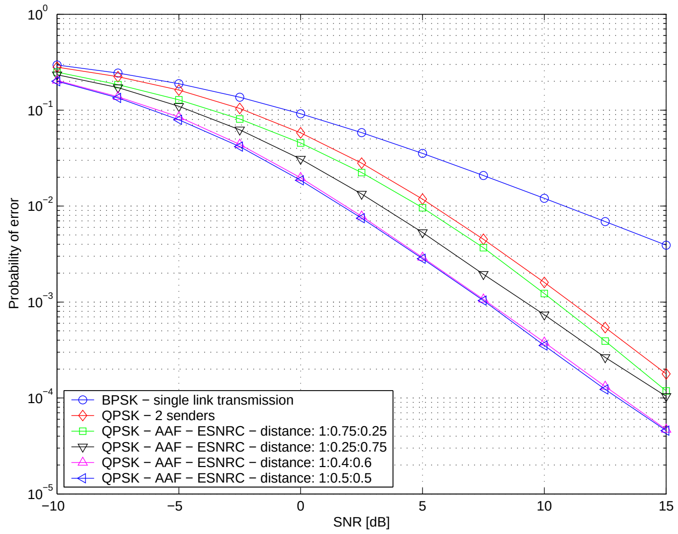

Fig. 3. A big benefit results when the relay is located between the sender and the destination.

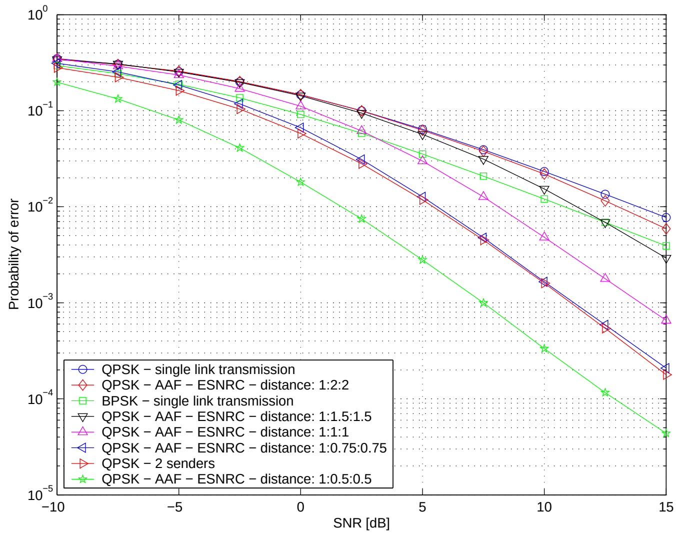

Fig. 4. Shows the effect of increasing the distance of the relay to the sender and the destination.

that would have been detected wrongly is just 'a little bit' wrong. When this symbol is amplified before sent to the destination, it has on average much less energy than the correct symbol coming directly from the sender. There is now a high probability that the incorrect symbol will be corrected by the signal from the direct link, when combined at the destination.

It is obvious now, why the fixed ratio combining shows such a good performance. The direct link has on average the better quality than the multi-hop link, so it is sensible to weigh the direct link more by assuming that the multi-hop link is more susceptible to errors than the direct link. It also explains why the optimal ratio in the system using DAF is higher than the one using AAF. The DAF relay sends the wrongly detected symbols with the full power, so it takes much more to correct this wrong powerful symbol.

The ESNRC shows roughly the same performance in a AAF or DAF system. The DAF using system benefits a lot by analysing every single block. Using this combining method the big disadvantage of the wrongly detected symbol at the relay can be reduced. In the majority of the cases, when a symbol is wrongly detected by the relay, the multi-hop has a much poorer channel quality than the direct link, and therefore will not be considered at all.

It might be sensible to ask now, what the purpose is of making the effort at the relay station to decode and re-encode the data, when there is no benefit at all doing that. As mentioned in Sec. III-B an error correcting code can be added to correct wrongly detected symbols at the receiver. This is, as seen before, crucial to get a good performance in a DAF system.

B. *Moving the Relay*

So far, the three stations were positioned equidistantly and therefore the three channels had all the same average signal-to-noise ratio. In this section the effect is shown when the relay station is moved. Notice, in the following figures the x-axis denotes always the SNR of the direct link, which can be assumed (without any loss of generality) to have a length of one.

The optimal arrangement is shown in Fig. 3 where the relay is placed right between the other stations. This shows the full potential of the used diversity arrangement. The performance is as good as the two senders arrangement. It is a little bit surprising that the resulting performance is not symmetric. The optimal position for the relay is right in the middle or slightly closer to the base station. This tendency to the base station is due to the fact that the noise received at the relay is amplified and therefore should be minimised. Another point that should be paid attention to is the huge benefit compared to the BPSK direct link. To achieve a BER of about 10^{-2} the SNR is up to eight decibels less than using only a direct link transmission.

Normally there is no relay station available just between the sender and the destination. But how close a mobile station has to be to act as a valuable candidate as a relay station? This is shown in Fig. 4.

The first thing that attracts attention is how fast the performance degrades when the distance of the relay increases. By increasing the distance by fifty percent, the resulting performance is roughly the same as the one for a two sender system, which is about three decibels less than the one of the optimal position. The position of the relay, where all three stations are equidistant, results in another 2.5 decibel loss in the system performance. This equidistant arrangement still shows an advantage compared to the BPSK single link transmission. This performance degredation continues, when the distance of the relay is increased further. Another fifty percent, results in a situation where there is no useful advantage anymore using the relay link. Notice that the higher diversity level can still be recognised.

VI. CONCLUSIONS

This work has shown the possible benefits of a wireless transmission using cooperative diversity to increase the performance. The diversity is realised by building an ad-hoc network using a third station as a relay. The AAF protocol has shown a better performance than the DAF protocol whatever combining method was used at the receiver. But it must be considered that no error correcting code was added to the transferred signal.

The choice of combining method has a big effect on the error rate at the receiver. When AAF is used at the relay station the easy to implement *Equal Ratio Combining* (ERC) shows some benefits compared to the single link transmission. If possible the *Fixed Ratio Combining* (FRC) should be used. This only needs knowledge of the average channel quality, and shows a much better performance than the ERC. If knowledge of the current state of the channel quality is available more sophisticated combining methods can be used. The *Enhanced Signal-to-Noise Ratio Combining* (ESNRC) has shown a very good performance considering that a rough approximation of the current channel quality is sufficient.

The location of the relay is crucial to the performance. The best performance was achieved when the relay is at equal distance from the sender and the destination or slightly closer to the former. In general the relay should not be to far from the line between the two stations.

ACKNOWLEDGEMENTS

Andreas Meier gratefully acknowledges to everybody supporting his work. Especially to Dr. John Thompson for supporting in technical issues and to Dave and Brian for their linguistic help.

REFERENCES

[1] John G. Proakis, *Digital Communications*, McGraw-Hill, 4th edition (international), 2001.

[2] J. Nicholas Laneman, David N. C. Tse, and Gregory W. Wornell, *Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behaviour*, 2002.

[3] M. Yuksel and E. Erkip, *Diversity in Relaying Protocols with Amplify and Forward*, in Proceedings of 2003 GLOBECOM Communication Theory Symposium, San Francisco, December 2003.

APPENDIX A: ESTIMATION OF THE SNR

A. Estimate SNR using AAF

Using AAF, the received signal from the relay is

$$y_{r,d} = h_{r,d}x_r + z_{r,d} = h_{r,d}\beta(h_{s,r}x_s + z_{s,r}).$$

The received power will then be

$$E[|y_{r,d}|^2] = \beta^2|h_{r,d}|^2(|h_{s,r}|^2\xi + 2\sigma_{s,r}^2) + 2\sigma_{r,d}^2,$$

so the SNR of the one relay multi-hop link can be estimated as

$$\text{SNR} = \frac{\beta^2|h_{s,r}|^2|h_{r,d}|^2\xi}{\beta^2|h_{r,d}|^2 2\sigma_{s,r}^2 + 2\sigma_{r,d}^2}. \tag{10}$$

B. Estimate SNR using DAF

To calculate the SNR of a multi-hop link using DAF, first the BER of the link is calculated which can then be translated to an equivalent SNR.

The BER of a single link is given in (2). The BER over a one relay multi-hop link can then be calculated as

$$\text{BER}_{s,r,d} = \text{BER}_{s,r}(1 - \text{BER}_{r,d}) + (1 - \text{BER}_{s,r})\text{BER}_{r,d}.$$

To calculate the SNR, the inverse function of (2) is used. For a BSPK modulated Rayleigh faded signal this will be

$$\text{SNR} = \frac{1}{2}\left[Q^{-1}(\text{BER})\right]^2. \tag{11}$$

For a QPSK modulated signal this will change to

$$\text{SNR} = \left[Q^{-1}(\text{BER})\right]^2. \tag{12}$$